MA114 Summer 2018
Worksheet 11 – Alternating Series – 6/26/18

1. (a) Let $a_n = \frac{n}{3n+1}$. Does $\{a_n\}$ converge? Does $\sum_{n=1}^{\infty} a_n$ converge? a) $\{a_n\} \to \frac{1}{3}$, so $\sum a_n$ diverges by Div Test.
   (b) Give an example of a divergent sequence $\sum_{n=1}^{\infty} a_n$ where $\lim_{n\to\infty} a_n = 0$. b) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$
   (c) Is there any example of a convergent sequence $\sum_{n=1}^{\infty} a_n$ where $\lim_{n\to\infty} a_n \neq 0$? c) No, by the Div Test.
   (d) Suppose we have an alternating series $\sum_{n=1}^{\infty}(-1)^{n+3}a_n$, where $a_n \geq 0$. Is it possible that the series diverges? d) Yes, $\sum_{n=1}^{\infty} (-1)^{n+3}$.

2. Decide whether the Alternating Series Test can be used to show that the following series converge. If it cannot, explain why and if possible use a different test to determine whether the series converges or not.

AST shows convergence (a), (b)

(a) $\sum_{n=1}^{\infty}(-1)^n \frac{\sqrt{n}}{1+2n}$ $\lim_{n\to\infty} \frac{\sqrt{n}}{1+2n} = \lim_{n\to\infty} \frac{1}{4\sqrt{n}} = 0$, $f'(x) = \frac{\frac{1}{2\sqrt{x}}(1+2x) - 2\sqrt{x}}{(1+2x)^2} = \frac{1+2x-4x}{2\sqrt{x}(1+2x)^2} = \frac{1-2x}{2\sqrt{x}(1+2x)^2} < 0$ if $x \geq \frac{1}{2}$. So $\{b_n\}$ is decreasing.

(b) $\sum_{k=2}^{\infty}(-1)^{k+1} \frac{1}{\ln(k)}$ $\lim_{k\to\infty} \frac{1}{\ln k} = 0$, $\frac{1}{\ln(k+1)} < \frac{1}{\ln(k)}$

(c) $\sum_{m=2}^{\infty} \frac{3^m}{4^m + 5^m}$ AST does not apply b/c not alternating. Use CT/LCT to $\frac{3^m}{4^m}$

(d) $\sum_{n=2}^{\infty}(-1)^n \frac{n}{\ln(n)}$ AST does not apply b/c $\lim_{n\to\infty} \frac{n}{\ln(n)} = \infty$. Divergence Test.

(e) $\sum_{n=1}^{\infty}(-1)^n \frac{\cos^2(n)}{n}$ AST does not apply b/c $\frac{\cos^2(n+1)}{n+1}$ is not always $\leq \frac{\cos^2(n)}{n}$. Need more tests.

(f) $\sum_{i=1}^{\infty} \left(\frac{-5}{18}\right)$. AST does not apply b/c not alternating. Divergence Test.

3. Estimate the series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n}{8^n}$ correct to three decimal places, i.e. so that the absolute error is at most 0.0005.

The series obeys the conditions of AST: $\lim_{n\to\infty} \frac{n}{8^n} = 0$, $\frac{n+1}{8^{n+1}} = \frac{n}{8^{n+1}} + \frac{1}{8^{n+1}} \leq \frac{8n}{8^{n+1}} = \frac{n}{8^n}$.

So $\left| \sum_{n=1}^{N} (-1)^{n-1} \frac{n}{8^n} - \sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{8^n} \right| \leq \frac{N+1}{8^{N+1}}$.

The smallest N so that $\frac{N+1}{8^{N+1}} \leq 0.0005$ is $N=4$.

So our estimate is $\frac{1}{8} - \frac{2}{64} + \frac{3}{512} - \frac{4}{4096} = \boxed{0.0986328125}$

To 3 decimal places: $\boxed{0.099}$